**Towards Fairer Centroids in k-means Clustering**

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**Abstract**

There has been much recent interest in developing fair clus- tering algorithms that seek to do justice to the representa- tion of groups defned along sensitive attributes such as *race* and *sex*. Within the centroid clustering paradigm, thesealgo- rithms are seen to generate clusterings where different groups are disadvantaged within different clusters with respect to their representativity, *i.e.*, distance to centroid. In view of this defciency, we propose a novel notion of *cluster-level centroid fairness* that targets the representativity unfairness borne by groups within each cluster, along with a metric to quantify the same. Towards operationalising this notion, we draw on ideas from political philosophy aligned with consideration for the worst-off group to develop *Fair-Centroid*; a new clustering method that focusses on enhancing the representativity of the worst-off group within each cluster. Our method uses an iter- ative optimisation paradigm wherein an initial cluster assign- ment is refned by reassigning objects to clusters such that the worst-off group in each cluster is beneftted. We compare our notion with a related fairness notion and show through ex- tensive empirical evaluations on real-world datasets that our method signifcantly enhances cluster-level centroid fairness at low impact on cluster coherence.

**Introduction**

Fairness in clustering has seen much scholarly activity in recent times (Chhabra, Masalkovaite˙, and Mohapatra 2021). Most of these endeavours, starting with Chierichetti et al. (2017), target proportional representation of sensitive groups – such as those defned on *race* and *sex* – within each cluster; this is often referred to as *group fairness* (Dwork et al. 2012). Such representational fairness of groups may be seen as the application of the notion of proportional representation, *aka* statistical parity (Besse et al. 2022), within clustering.

Within the *centroid clustering* paradigm pioneered by classical algorithms such as k-means, only ensuring pro- portional representation of groups within each cluster may be insuffcient depending on how subsequent decisions are made. Where decisions are to be made solely on cluster membership, this notion of group fairness would be appro- priate. In centroid clustering however, each cluster is addi- tionally characterised by a *centroid*, making it distinct from

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other clustering paradigms. Here, a data object’s proximity to its cluster centroid is key in determining the clustering quality; an object that is close to its centroid would be better represented within the clustering than another object that is farther away. This additional characteristic opens up a new avenue of unfairness in terms of how well a cluster’s cen- troid represents the objects in that cluster. Such a considera- tion is critical in decision-making scenarios where centroids are central such as facility location (*e.g.*, polling sites (Chen et al. 2022; Brady and McNulty 2011)) and summarisation.

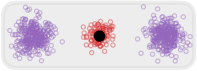
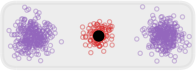
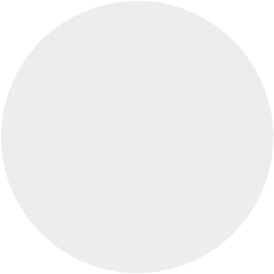
Towards this, P and Abraham (2020) seek to deepen the uniformity of every object’s distance-to-centroid, dubbed *representativity fairness*, regardless of sensitive group mem- bership; this falls within the scope of *individual fair- ness* (Dwork et al. 2012). Abbasi, Bhaskara, and Venkata- subramanian (2021) and Ghadiri, Samadi, and Vempala (2021) extend this notion to sensitive groups – they con- sider the mean representativity across objects within each group, and target equity along such aggregates. For example, in a social profle clustering scenario, the mean distance-to- centroid of female profles should be as close as possible to that of males. This notion, called *fair* k*-means*, targets over- all group fairness, *i.e.*, groups be treated fairly in the cluster- ing regardless of cluster membership. Against this backdrop, we note two issues with fair k-means’ approach to represen- tativity aggregations across groups: (i) representativities are not comparable across clusters, and (ii) poor representativ- ityin one cluster could negatively impact other clusters. We examine these using illustrative examples1 .

*First*, representativity is quantifed as an object’s distance to its cluster centroid. This construction does not yield well to cross-cluster comparisons when clusters are of different sizes as Figure 1 illustrates. The *purple* object here has bet- ter representativityin the large cluster than the*red* one, and vice versa. When conditioned on cluster size, the same dif- ference in representativity is almost insignifcantin the large cluster but very consequential in the small one, causing the *red* group to have a better overall representativity than the *purple* group. In sharp contrast to such intuitive judgement, the simple aggregation of representativity misleadingly puts both groups on an equal footing.

1The data objects in Figures 1–3 are two dimensional, shown on the xy-plane. Each grey region depicts a cluster.

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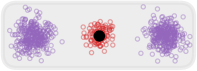
60



80

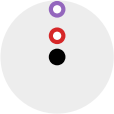
80

80

C0

C0

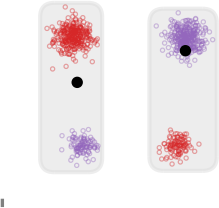
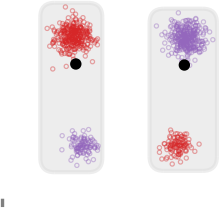
C0

40

60

60

60

20

40

40

40

C1

0

0 20 40 60 80

20

20

20

Figure 1: Representativity across differently sized clusters. The *red* and *purple* circles are two objects of distinct groups, and the distance between these two objects is the same in both clusters. Solid black circles depict centroids.

C1 C2

C1 C2

C2

0

0

0

0 20 40

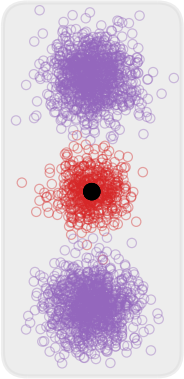
0 20 40

(a) k-means

0 20 40

(b) fair k-means

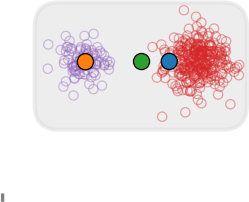
(c) fair centroids

Figure 3: Poor representativityin one cluster (C0 ) affecting cluster membership far away. Solid black circles depict cen- troids. C0’s centroid is unchanged across the three methods.

C1 ’s centroid  k-means

 fair k-means  fair centroids

40

clustering shown in Figure 3c would target fairness in indi- vidual clusters, preventing the unfairness experienced in one cluster from affecting other clusters.

20

**Our Contributions** We introduce *cluster-level centroid fairness* (CCF) – a novel formulation of group fairness ex- tending the notion of representativity fairness along sensitive groups in the data. Our notion rectifes the identifed cross- cluster representativity aggregation issues of fair k-means, thus generating fairer centroids. In the interest of empiri- cally comparing this notion with fair k-means, we opera- tionalise the former through a novel fair clustering method – *Fair-Centroid*, and illustrate through extensive empirical evaluations that our method is able to achieve high degrees of fairness on appropriate evaluation metrics.

C1

C0

0

0 20 40 60 80

Figure 2: Poor representativityin one cluster (C0 ) affecting another’s (C1’s) centroid. The two groups are shown in *red* and *purple*. C0’s centroid,in black, is at the best position.

*Second*, simply aggregating representativities across clus- ters could cause a group’s poor representativityin one clus- ter to negatively impact another cluster. In Figure 2, *purple*’s poor representativityin C0 is compensated by placing C1’s centroid (in orange) closer to *purple*, thus improving its rep- resentativity but worsening *red*’s representativityin C1 . Yet, *purple* happens to be the most disadvantaged group across the clustering. If C1 is looked at in isolation, this centroid heavily disadvantages the group that is not the worst-off overall (*red*). This is especially unjustifable when the two clusters are distant and possibly unrelated, thus begging the question – *is it fair and justifed to disadvantage a group in one cluster just because it is advantaged in another clus- ter?* Instead, a fair centroid for C1 (in green) would be one that considers each cluster independently, appreciating that clusters could have different worst-off groups. Indeed, hav- ing different worst-off groups in different regions in the data is especially prevalent in geographical data where racially segregated regions is common (*e.g.*, Abbasi et al. (2023)). Furthermore, poor representativity in one cluster could also affect cluster membership, as Figure 3 shows. Notice here that fair k-means compensates for *purple*’s poor representa- tivityin C0 by choosing C2’s centroid close to *purple* (Fig- ure 3b), even closer than k-means (Figure 3a), making fair k-means less fair than k-means for C2 . Apparently, fair k- means’ paradigm of cross-cluster aggregation of representa- tivities allows such compensatory effects to occur, thus act- ing as a veneer to conceal or exacerbate (as is the case in C2 ) potentially deep levels of intra-cluster unfairness. A fairer

**Related Work**

The group fairness notion targetting proportional represen- tation of sensitive groups within clusters was pioneered by Chierichetti et al. (2017). Since then, this stream has diversi- fed into considering paradigms such as spectral (Wang et al. 2023) and hierarchical clustering (Knittel et al. 2023). Vari- ants of group fairness notions such as capped representa- tion (Ahmadian et al. 2019) and fair labelled clustering (Es- maeili et al. 2022) have been explored. Orthogonal to these, Hotegni, Mahabadi, and Vakilian (2023) focus on the repre- sentation of groups in the set of centroids.

Along the facet of centroid proximity, P and Abraham (2020) optimise for the uniformity of representativity across all objects, while Abbasi, Bhaskara, and Venkatasubrama- nian (2021) and Ghadiri, Samadi, and Vempala (2021) target equitable overall group representativity. Better approxima- tion algorithms (Makarychev and Vakilian 2021; Goyal and Jaiswal 2023) and generalisations (Chlamt, Makarychev, and Vakilian 2022; Gorantla et al. 2023) for the latter have since been designed. Buet-Golfouse and Utyagulov (2022) look at this problem within the framework of fair gener- alised low-rank models, and Chhabra, Singla, and Mohap- atra (2022) proposed a dataset augmentation approach.

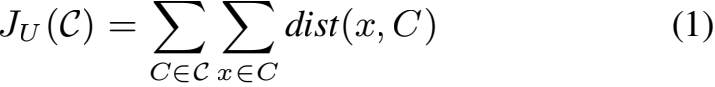
**Positioning Our Work** Our notion of cluster-level cen- troid fairness is a new conceptualisation of fairness unex-

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plored in previous work. Our use of cluster-level group rep- resentativity fairness quantifcation as an intermediate level between individual and dataset-level group fairness makes it distinct from previous work on both fairness streams.

**Background**

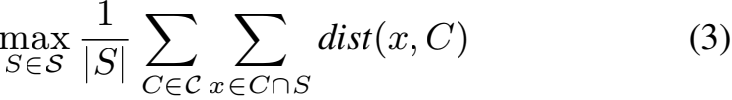
We briefy outline the formulation of the popular k-means clustering problem (MacQueen 1967). Let X be a set of re- lational data objects to be partitioned into k clusters denoted by C. Lloyd’s heuristic for k-means (Lloyd 1982) uses an EM-style framework to optimise for the objective JU



where *dist*(x, C) is the squared Euclidean distance of object x to the centroid µC of cluster C given by

*dist*(x, C) = ||x − µC || 2 (2) Thus, *dist*(x, C) quantifes the representativity of x with re- spect to C. Notice that the objective relates to a given cluster assignment C; the EM-style optimisation starts with a given cluster assignment, and iterativelyrefnes the cluster assign- ment and centroids to minimise the objective in Equation 1.

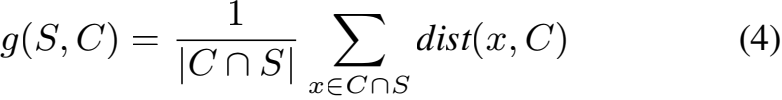
As a generic clustering problem, k-means ignores sensi- tive group membership – it optimises for the sum of repre- sentativities across all objects. This may be regarded as a Benthamite utilitarian objective that seeks *the greatest good for the greatest number* (Bentham 1996). Abbasi,Bhaskara, and Venkatasubramanian (2021) and Ghadiri, Samadi, and Vempala (2021) note that this can result in centroids repre- senting groups differently, often favouring one over others, thus having implications when centroid fairness is central. To mitigate the disparity in representativities of groups, they independently propose the fair k-means notion. Let each ob- ject in X belong to one or more of several groups S (*e.g.*, *white*, *asian*, *etc.*) defned across a sensitive attribute S (*e.g.*, *race*). The fair k-means objective optimises for the overall (dataset-level) worst-off group by minimising



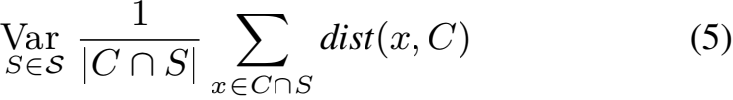
Note that this objective’s averaging property allows a group’s poor representativityin one cluster to be offset by good representativities in other clusters (as seen for the *red* group in Figures 2 and 3b). Furthermore, the same group may not be the worst-off across all clusters, as this objective assumes. Thus,optimising for the overall worst-off may not augur well for clusters whose worst-off is not the worst-off overall. Towards this, we focus on mitigating group unfair- ness at the cluster-level rather than at the dataset-level.

**Cluster-level Centroid Fairness**

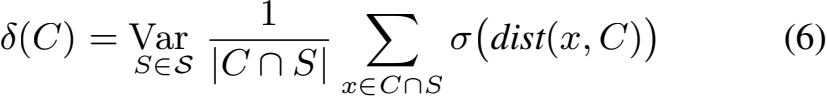
Our notion of cluster-level centroid fairness (CCF) targets to minimise the disparity in the representativities of groups within each cluster. We assume a single sensitive attribute S. Consider a cluster C within a cluster assignment C. The representativity of group S within C, denoted by g(S, C), is



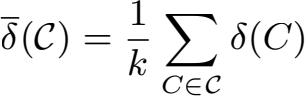
Thus for each cluster C, we obtain a *representativity vector* with one component per group. *We say that a cluster is fair if its representativity vector is uniform, and a clustering is fair if all clusters in it are fair.* In view of this novel formulation of cluster-level group representativity fairness, we develop a metric towards quantifying adherence to the notion. Ac- cordingly, we capture the representativity disparity between the groups within cluster C simply by taking the variance in their representativities, given by



Our choice of variance for quantifying group fairness is sim- ilar to P and Abraham (2020)’s quantifcation of individual fairness which is based on egalitarianism, our focus being at the group-level (*e.g.*, gender equality, racial equality). Here, the ideal value 0 is achieved when every group in the cluster has the same representativity. Now, when aggregating across clusters of different sizes, this formulation can allow vari- ances in large clusters to conceal variances in small ones. Towards allowing a fair basis of aggregation, we standard- ise the representativities of objects in each cluster separately regardless of group membership,*i.e.*, the objects’ represen- tativities within a cluster would be transformed to have zero mean and unit variance. With σ referring to this transforma- tion, we thus compute the *cluster disparity* δ(C) as



To arrive at a single measure for the clustering C, we aggre- gate this across clusters as

 (7)

where k= | C | is the number of clusters. The *average cluster disparity* δ(C), being an average of variances, evaluates to a non-negative value, with a lower value indicating a smaller disparity between the groups’ representativities in individ- ual clusters, and consequently a fairer clustering. This met- ric thus quantifes our notion of CCF. Observe that δ(C) does not capture utility in any way, and thus provides no indica- tion on the quality of the clustering.

**Considerations** Note that being underpinned by the con- cept of representativity fairness which is inherently centroid dependent, CCF is designed to work with centroid-based clustering algorithms; non-centroid paradigms are thus be- yond our scope. Additionally, given a plethora of fairness notions, the one to be applied in a specifc context needs to be deliberated and situated within the nuances of the sce- nario. For example, CCF is highly appealing for facility lo- cation in geographically segregated regions where different regions may have different worst-off groups.

**Fair-Centroid**

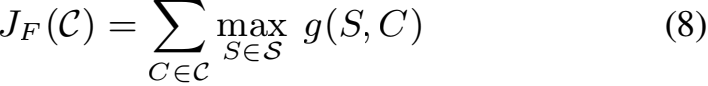
In the interest of comparing CCF with other fairness and utilitarian notions, we operationalise CCF through Fair- Centroid, a novel fair clustering method. We describe our objective function followed by the optimisation framework.

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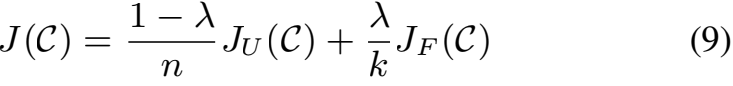
**Objective Function**

Recall that CCF targets to enhance the uniformity of the representativity vector for each cluster. This, as Abbasi, Bhaskara, and Venkatasubramanian (2021) note, can be triv- ially achieved by having a poor representativity for all sen- sitive groups thus making the resulting clustering of poor utility. Instead, we take cue from contemporary theories in political philosophy and focus on mitigating the represen- tativity loss (*i.e.*, Equation 4) experienced by *the worst-off group within each cluster*. In contrast to weighted formula- tions for groups within clusters, this formulation espouses the ethos across several popular philosophical theories in- cluding *concern for the most vulnerable* within the famed *difference principle* (Freeman 2018) of distributive justice due to John Rawls2 , and pervades eastern ideals (*e.g.*, Gand- hian thought (P and Abraham 2020)). Similar minimax for- mulations have also been explored for fairness in classif- cation (Martinez, Bertran, and Sapiro 2020), dimensionality reduction (Samadi et al. 2018), and clustering (Equation 3). Thus, our goal here is to generate a coherent clustering (the singular focus of algorithms such as k-means) where addi- tionally, the *representativity of the worst-off group within each cluster* is improved as much as possible. Given that the utilitarian consideration of *cluster coherence* (that clas- sicalk-means also targets to optimise) would be in apparent tension with the *cluster-level group representativity fairness* consideration, we look to deepen the latter at as little detri- ment to the former as possible.

Given our intent of improving the representativity of the worst-off group within each cluster, we model our fairness objective JF , which targets fairer centroids, as the aggregate of representativities of the worst-off group in each cluster



This *fair centroid* objective captures the fairness ethos es- poused by CCF, albeit using signifcantly different method- ology. We also optimise for the utilitarian k-means objective JU (Equation 1). Our overall objective function J is thus



where n= |X| is the number of objects to be clustered and 0≤λ≤1 is a hyperparameter that controls the trade-off be- tween utility and fairness. To prevent one objective from dominating, we rescale; JU being a summation over all ob- jects in the dataset is divided by n, and since JF being a summation over all clusters is divided by k.

**Optimisation Framework**

The parameters to be estimated in our objective function (Equation 9) are the cluster assignment C and the set of cen- troids {µC }. Towards this, we follow the same EM-style it- erative procedure as in Lloyd’s heuristic fork-means (Lloyd 1982) that alternates between (i) estimating the cluster as- signment keeping the set of centroids fxed (E-step), and (ii) estimating the set of centroids keeping the cluster as- signment stationary (M-step). We now describe the E and M-steps within our optimisation framework.

2<https://en.wikipedia.org/wiki/John> Rawls

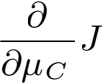
|  |  |
| --- | --- |
| **Algorithm 1** EstimateAssignment | |
| 1: **for all** x ∈ X **do** | |
| 2: | **for all** C′ ∈ C **do** |
| 3: | Obtain C′ using Equation 10 |
| 4: | **if** J(C′ ) < J(C) **then** |
| 5: | C ← C′ |
| **E-step: Estimating the Cluster Assignment** Given the set of centroids {µC }, we need to assign objects to clusters that minimise our objective function (Equation 9). Towards this, given the complexity of the objective function, we per- turb the existing cluster assignment by making a single pass through all objects and greedily reassigning them to clusters such that the value of the objective function decreases. Thus, if an object x is reassigned from cluster C to cluster C′ , the new cluster assignment C′ is  C′ = C \ {C, C′ } ∪{C \ {x}, C′ ∪ {x}} (10)  Algorithm 1 outlines our greedy approach. Within each E- step, this entails trying out O(nk) cluster reassignments, k−1 per object. Note that the change between J(C′ ) and J(C) where C andC′ differ in the membership of a single ob- ject can beeffciently computed without a full dataset-wide estimation, similar in spirit to what is outlined by Abraham, P, and Sundaram (2020, §4.2.1)3 .  **M-step: Estimating the Cluster Centroids** Our goal here is to estimate the set of centroids {µC } that minimises the objective function in Equation 9 while keeping the cluster assignment C fxed. We use the gradient descent framework where the intent is to move along the negative gradient. Since the max operator in Equation 8 is not differentiable, we follow P and Abraham (2020) in using a weighted Log- SumExp as a differentiable approximation (Buet-Golfouse andUtyagulov 2022)    where ϕ∈R+ is a large enough positive constant that ampli- fes the signifcance of the largest number. Substituting this in Equation 8 gives us a differentiable approximation for JF    The derivative of J with respect to µC is3    where  JU = −2 ×  (14)  (15) | |

3For more details, see the extended version at <https://doi.org/> 10.48550/arXiv.2212.14467

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|  |  |
| --- | --- |
| **Algorithm 2** EstimateCentroids | |
| 1: **repeat** | |
| 2: | **for all** C ∈ C **do** |
| 3: | Compute J using Equations 13–16 |
| 4: | Perform an update for µC using Equation 17 |
| 5: | **until** convergence |

w(S, C) = exp (ϕ × g(S, C)) (16) Much like the E-step, regularities in the construction of Equation 13 allow for effcient incremental gradient compu- tation. Equations 13–16 are used to iteratively update the set of cluster centroids {µC } within the gradient descent frame- work as

µC ← µC − η  (17)

where η∈R+ is the learning rate. The M-step is outlined in Algorithm 2. Convergence is declared once the Frobenius norm of the difference in centroids in successive iterations is within a specifed relative tolerance hyperparameter τ∈R+ , similar to the k-means implementation4 in scikit-learn (Pe- dregosa et al. 2011), a popular Python machine learning li- brary. This is complemented by another stopping condition where we break the iterative process if the Frobenius norm has not decreased in the last 10 iterations. To summarise, within each M-step, the cluster centroids are iteratively up- dated until convergence using the update in Equation 17.

**Stopping Condition** Our EM procedure stops when at least one of the following holds in successive EM-steps: (i) the cluster assignment is unchanged, or (ii) the Frobe- nius norm between the centroids in the two steps is within the specifed tolerance hyperparameter τ . Additionally, fol- lowing Ghadiri, Samadi, and Vempala (2021), we halt if the above stopping conditions are not met within 200 iterations.

**Empirical Evaluation**

Since CCF is a new notion with no associated method, our focus is more on evaluating CCF against utilitarian and fair- ness notions rather than rigorously evaluating the perfor- mance3 of methods that operationalise these notions. We start by describing the experimental setup, followed by re- sults and analyses5 . Our implementations6 are in Python 3, and all experiments were run on the Kelvin2 cluster7 (2GHz AMD processor and 5GB RAM).

**Experimental Setup**

**Datasets** Our datasets (Table 1) are based on the pub- licly available Adult (Becker and Kohavi 1996) and Cred- itCard (Yeh 2016) datasets. Both contain people data and

4<https://scikit-learn.org/stable/modules/generated/sklearn>. cluster.KMeans.html

5For all fguresin this section, they-axes differ for the datasets, and lower values on they-axes are better.

6<https://github.com/stanleyts/faircentroid> 7<https://www.ni-hpc.ac.uk/Kelvin2/>

|  |  |  |  |
| --- | --- | --- | --- |
| dataset/  sensitive attribute | sensitive groups | non-sensitive attributes | objects |
| Adult/sex Adult/race | 2  5 | 26 | 46033 |
| CreditCard/SEX | 2 | 77 | 30000 |

Table 1: Datasets used

include sensitive information such as *race* and *sex* (which are protected from being the basis of discrimination by laws, *e.g.*, UK’s Equality Act 20108 ), making them popu- lar for benchmarking in the algorithmic fairness commu- nity (Chhabra, Masalkovaite˙, and Mohapatra 2021; Fabris, Silvello, and Susto 2022; Le Quy et al. 2022). We consider sex and race as the two sensitive attributes for Adult, and SEX as the sensitive attribute for CreditCard3 .

**Baselines** CCF being a novel fairness formulation that has been hitherto unexplored in literature, there are no suit- able state-of-the-art baseline clustering methods to compare against. Abbasi, Bhaskara, and Venkatasubramanian (2021) and Ghadiri, Samadi, and Vempala (2021), being based on group representativity fairness, are related but optimised for a fairness objective different from ours. In the inter- est of comparing CCF with existing utilitarian and fairness notions, we benchmark our Fair-Centroid method against methods that operationalise these notions: Lloyd’s heuristic fork-means and Fair-Lloyd (Ghadiri, Samadi, and Vempala 2021) for fair k-means. Since the available implementation of Fair-Lloyd9 only handles binary sensitive attributes, we use an in-house implementation3 with gradient descent in- stead of line search for centroid computation. We do not compare with Abbasi, Bhaskara, and Venkatasubramanian (2021), theirs being more suited for the facility location problem rather thank-means clustering.

**Parameter Confguration** In all experiments, unless specifed, we set λ=0.5 to give equal importance to the utili- tarian and fairness objectives in our method. For trends on k, we take values fork in the range 3 to 12 with a step size of 1. For gradient computation of the objective functions, we set ϕ=103 and η=10–3 . For convergence in Fair-Centroid’s M-step, we set τ=10–4 similar to scikit-learn’s k-means. All numbers reported are averaged over 100 runs with initial centroids estimated using k-means++ (Arthur and Vassilvit- skii 2007).

**Results**

**Fairness vs Utility** It is widely accepted that increase in fairness almost always causes decrease in utility; it would be of interest to look at the CCF gains obtained due to Fair-Centroid and the corresponding loss in utility. Here, we quantify CCF with our average cluster disparity metric (Equation 7), and utility with the k-means objective10 . Fig-

ures 4 and 5 show trends for the trade-off across different 8<https://www.gov.uk/discrimination-your-rights>

9<https://github.com/samirasamadi/SociallyFairKMeans>

10We rescale the k-means objective, *i.e.*, divide Equation 1 by n.

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| k-means objective average cluster disparity | × 10-2  2 .0  1 .5  1 .0  0 .5  0 .0  2   |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  |  |  |  |  |  | |  |  |  |  |  |  | |  |  |  |  |  |  | |  | Ll | oyd |  |  |  | |  | Fa Fa | ir-L ir-C | loy ent | d  roid |  | | 0 2 4 6 8 10 1 | | | | | |   k | 2 .5  2 .0  1 .5  1 .0  0 .5  0 .0 | × 10-2 | | × 10-3  5   |  | | --- | |  |   4  3  2  1  0  0 2 4 6 8 10 12  k |
| 0 2 4 6 8 10 12 | |
| k  (b) Adult/race | |
| (a) Adult/sex |
| (c) CreditCard/SEX |
| Figure 4: Average cluster disparity (δ ) across k | | | | |
| 3 .0  2 .5  2 .0  1 .5  1 .0  0 .5  0 .0  2   |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  |  |  |  |  |  | |  |  |  |  |  |  | |  |  |  |  |  |  | |  |  |  |  |  |  | |  |  |  |  |  |  | |  | Ll Fa | oyd ir-L | loy | d |  | |  | Fa | ir-C | ent | roid |  | | 0 2 4 6 8 10 1 | | | | | |   k | 3 .0  2 .5  2 .0  1 .5  1 .0  0 .5  0 .0 | 0 2 4 6 8 10 12 | 10  8  6  4  2  0  0 2 4 6 8 10 12  k | |
| k  (b) Adult/race |
| (a) Adult/sex |
| (c) CreditCard/SEX | |
| Figure 5: k-means objective (JU /n) across k | | | | |

values of k. Fair-Centroid consistently and signifcantly re- duces the unfairness over Lloyd (Figure 4) at the cost of a comparably small increase in the utilitarian objective (Fig- ure 5). Fair-Lloyd does not perform as well on this fairness metric, sometimes even worse than Lloyd. Thus, optimising for fair k-means, as Fair-Lloyd targets, provides an impres- sion of overall fairness but conceals unfairness at a deeper level of individual clusters. In contrast, Fair-Centroid beats the baselines on our cluster-level group fairness metric,*i.e.*, average cluster disparity. From Figure 4 we infer that fair k- means, while being conceptually similar to CCF, advances representativity fairness in a way that is empirically anti- thetical to the consideration of the worst-off group in each cluster. Simply put, Fair-Lloyd (which optimises for fair k- means) nudges the clustering away from Lloyd to confgu- rations in a different direction than Fair-Centroid. Figure 4 thus provides empirical evidence in favour of our motivation that fair k-means may not be considerate to the worst-off group in individual clusters.

**Fair Centroid Objective vs Fair k-means** Towards com- paring the two notions of group representativity fairness, we look at how Fair-Centroid compares with the baselines on the two fairness objectives: (i) our fair centroid objec- tive11 (Equation 8), and (ii) fair k-means objective (Equa- tion 3). Note that the two focus on worst-off groups at dif- ferent levels – cluster-level vs overall. Figure 6 shows that Fair-Centroid improves the representativity of the worst-off groups over Lloyd, indicating that our method is moving in the right direction. Fair-Lloyd does not perform as well; this is expected as it is not designed for cluster-level group

11We rescale our objective, *i.e.*, divide Equation 8 by k.

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| fair centroid objective | 4  3  2  1  0 | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  |  |  |  |  |  | |  |  |  |  |  |  | |  | Ll | oyd |  |  |  | |  | Fa Fa | ir-L ir-C | loy ent | d  roid |  | | 0 2 4 6 8 10 1 | | | | | |   k  2  (a) Adult/sex | 4  3  2  1  0  0 2 4 6 8 10 12  k  (b) Adult/race | 140   |  | | --- | |  |   120  100  80  60  40  20  0  0 2 4 6 8 10 12  k  (c) CreditCard/SEX |

Figure 6: Fair centroid objective (JF /k) across k

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| fair k-means objective | 3 .5  3 .0  2 .5  2 .0  1 .5  1 .0  0 .5  0 .0 | |  | | --- | | Lloyd  Fair-Lloyd  Fair-Centroid |   0 2 4 6 8 10 12 | 4  3  2  1  0  0 2 4 6 8 10 12 | 10  8  6  4  2  0  0 2 4 6 8 10 12 |

k k k

(a) Adult/sex (b) Adult/race (c) CreditCard/SEX

Figure 7: Fair k-means objective across k

fairness. On the other hand, Fair-Centroid causes the overall worst-off group to have a worse representativity than Lloyd, as seen in Figure 7. While at frst glance this may seem to be a defciency of our method, any improvement in this group’s representativity would potentially result in another group that is the worst-off in some cluster being further dis- advantaged, as Figures 2 and 3 highlight. This would be un- acceptable in cases where more than one group are histor- ically disadvantaged (*e.g.*, Black and American Indians in case of *race*) and beneftting one would result in disadvan- taging the other. Evidently, optimising for one fairness ob- jective does not necessarily entail optimising for the other, *i.e.*, reducing overall unfairness could increase cluster-level unfairness, and vice versa.

**Controlling for Fairness** We study how λ – the trade-off between the fairness and utilitarian objectives JF and JU in Equation 9 – affects Fair-Centroid; we quantify fairness with average cluster disparity (Equation 7) and utility with the k- means objective10 . We set k=5 for Adult and k=4 for Cred- itCard, obtained using the elbow method (Thorndike 1953) on the k-means objective trend for Lloyd (Figure 5). We ex- periment with values for λ in the range 0.1 to 1 with a step size of 0.1 (λ=0 corresponds to Lloyd’s heuristic). Figure 8 shows that even for small λ, optimising for the fair centroid objective in addition to the utilitarian objective (*i.e.*, Equa- tion 9) substantially improves CCF over Lloyd’s heuristic. The corresponding loss in utility is comparably small as can be seen in Figure 9 thus indicating the effectiveness of Fair- Centroid in balancing the tradeoff between utility and CCF, making it useful in scenarios where fairness is to be ensured at little impact to utility.

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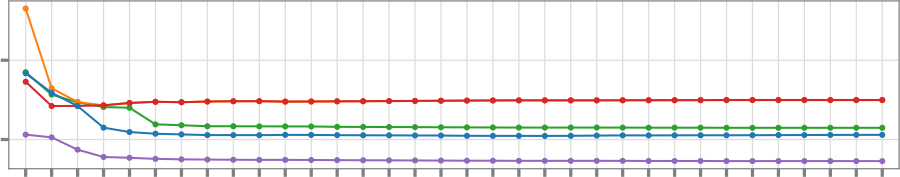
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| k-means objective average cluster disparity | × 10-2 × 10-3  × 10-2  1 .2   |  | | --- | |  |  |  | | --- | |  |  |  |  | | --- | --- | |  |  | | Lloyd  Fair-Centroid | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | |  |  |  |  |  | |  |  |  |  |  | |  |  |  |  |  | |  |  |  |  |  | |  |  |  |  |  | |  |  |  |  |  |   1 .75  1 .50  1 .25  1 .00  0 .75  0 .50  0 .25  1 .0  1 .0  0 .8  0 .8  0 .6  0 .6  0 .4  0 .4  0 .2  0 .2  0 .00 0 .0  0 .0 0 .2 0 .4 0 .6 0 .8 1 .0 0 .0 0 .2 0 .4 0 .6 0 .8 1 .0  0 .0   |  |  |  |  |  | | --- | --- | --- | --- | --- | |  |  |  |  |  | |  |  |  |  |  | |  |  |  |  |  | |  |  |  |  |  | |  |  |  |  |  |   0 .0 0 .2 0 .4 0 .6 0 .8 1 .0  人  人  (b) Adult/race  人  (c) CreditCard/SEX  (a) Adult/sex | |
| Figure 8: Average cluster disparity (δ ) across λ | |
| 2 .0  1 .5  1 .0  0 .5  0 .0   |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | |  | |  | |  |  |  | |  |  |  |  |  |  |  | |  | |  | |  |  |  | |  | |  | |  |  |  | |  | |  | |  |  |  | |  | | Lloyd Fair | | -Ce | ntro | id | | |  | | --- | |  | |  |   12  2 .0  10  1 .5  8  6  1 .0  4  0 .5  2  0 .0 0   |  |  |  |  |  | | --- | --- | --- | --- | --- | |  |  |  |  |  | |  |  |  |  |  | |  |  |  |  |  | |  |  |  |  |  | |  |  |  |  |  | |  |  |  |  |  |   0 .0 0 .2 0 .4 0 .6 0 .8 1 .0 0 .0 0 .2 0 .4 0 .6 0 .8 1 .0  人 人  (b) Adult/race (c) CreditCard/SEX |
| 0 .0 0 .2 0 .4 0 .6 0 .8 1 .0  λ  (a) Adult/sex |
| Figure 9: k-means objective (JU /n) across λ | |

**Disparity Trends** To qualitatively analyse Fair-Centroid’s behaviour, we look at how the sensitive groups’ representa- tivities (Equation 4) vary within clusters over iterations. As an illustration, Figure 10 shows the trends in the clusters on a run on the Adult/race dataset. Notice in the line plots that over iterations: (i) the disparities in the representativities of the best-off and worst-off groups decrease, and (ii) therepre- sentativities of the worst-off groups improve, as we set out to achieve. Also observe from the bar plot that both the worst- off groups and the quanta of representativities differ across clusters; these traits are captured by our CCF notion but not by fair k-means. Also note in the bar plot that the worst-off sensitive groups in the clusters change over iterations. While we observe these trends to generally hold across datasets and k, Figure 10 showcase all key points made in this paper.

**Computational Comparison** Fair-Centroid was found to need more iterations to converge than Lloyd and Fair-Lloyd due to its E-step yielding suboptimal cluster assignments; Lloyd and Fair-Lloyd use a closed-form expression to opti- mallyassign objects to clusters, thus requiring fewer itera- tions. In terms of runtime, Lloyd was the fastest, followed by Fair-Lloyd which takes longer due to the progressive re- fnement nature of its M-step, and Fair-Centroid taking the longest as both its E and M-steps use progressive refnement. To give the reader a sense of the runtime, Fair-Centroid with k=5 took ≈80 minutes on the Adult/sex dataset which has ≈46k data objects and 2 sensitive groups3 .

**Conclusion and Future Work**

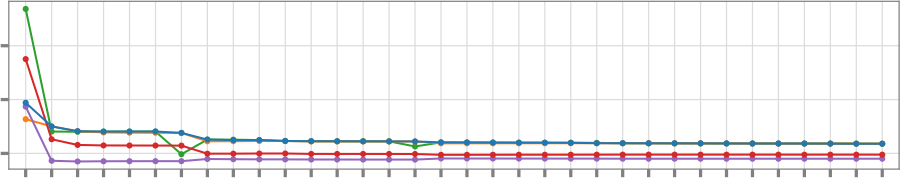
We introduced cluster-level centroid fairness (CCF); a new formulation of fairness for centroid clustering with respect



3

C0

2



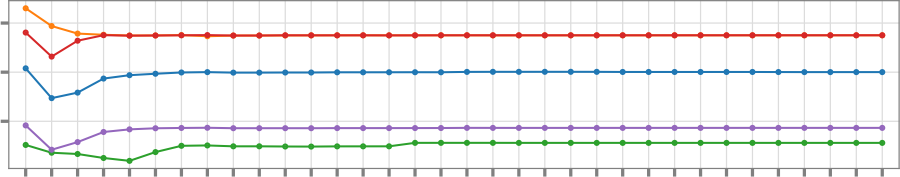
5

group representativity

C2 C 1

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3



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4

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worst-off sum

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| C0 |  |  | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
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| C2 |  |
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iteration

Figure 10: Trends in the 5 groups’ representativities for Adult/race (k=3,random state 60) in the 3 clusters due to Fair-Centroid across iterations. Each group is shown in a dif- ferent colour. The 3 line plots show the trends for all groups in the 3 clusters. The stacked bar plot shows the worst-off group in each cluster and its representativity. The height of each stacked bar is the sum of representativities of the worst- off group in each cluster, *i.e.*, the fair centroid objective.

to representativity of sensitive groups at the cluster-level. Against the backdrop of much recent enthusiasm in using representativity-oriented notions of fairness in clustering, we outlined issues engendered by simplistic aggregations of representativities across clusters, and how our notional- leviates such problems. We proposed Fair-Centroid; a new clustering method that iteratively improves the clustering to- wards our CCF notion. Given the novel form of our fairness notion, we introduced a new metric that captures disparity between the representativity of groups at the cluster level, accounting for the possibility of different groups being dis- advantaged within different clusters. Our experiments on the Adult and CreditCard datasets demonstrate our method’s ef- fectiveness in achieving high levels of cluster-level group representativity fairness at low impact to the popular utili- tarian cluster coherence metric used withink-means.

**Future Work** As Fair-Centroid is an initial attempt at op- erationalising CCF, designing effcient algorithms for CCF is a natural next step, *e.g.*, line search for centroid com- putation (see Ghadiri, Samadi, and Vempala (2021)), or a LP relaxation (see Abbasi, Bhaskara, and Venkatasubra- manian (2021)). Another direction would be to consider numeric sensitive attributes such as *age* – an attribute on which discrimination is well-understood within healthcare (*e.g.*, Dobrowolska et al. (2019)). In terms of clustering paradigms, it would be interesting to apply this work to other non-k-means centroid clustering formulations such as k-medoids (Park and Jun 2009) and fuzzy c-means (Bezdek, Ehrlich, and Full 1984).

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